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## More D-branes in plane wave spacetime

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### ABSTRACT

We present classical solutions of  $Dp$ -branes ( $p \geq 5$ ) in plane wave spacetime with nonconstant R-R 3-form flux. We also show the existence of a system of  $D3$ -branes in this background. We further analyze the supersymmetric properties of these branes by solving type II Killing spinor equations explicitly.

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# 1 Introduction

Study of string theory in plane wave background with flux has been the topic of intense discussion in recent past. It is known for quite sometime that pp-wave spacetime provides exact string theory backgrounds. These backgrounds are exactly solvable in lightcone gauge. Many of them are obtained in the Penrose limit (pp-wave limit) of  $AdS_p \times S^q$  type of geometry and in some cases are maximally supersymmetric [1, 2]. Strings in pp-wave background are also investigated to establish the duality between the supergravity modes and the gauge theory operators in the large R-sector of the gauge theory [3].

PP-wave background with nonconstant Ramond-Ramond (R-R) flux [4–8] gives an interesting class of supersymmetric pp-wave solutions in type IIB supergravity. The worldsheet theory corresponding to pp-waves with nonconstant R-R  $F_5$  flux is described by nonlinear sigma model which is supersymmetric and one can have linearly realized ‘supernumerary’ supersymmetries in these backgrounds [9]. PP-wave backgrounds supported by nonconstant R-R  $F_3$  fields, do not have, in contrast to their  $F_5$  counter part, supernumerary supersymmetries. These backgrounds provide, in general, examples of nonsupersymmetric sigma models [5] unless there exists some target space isometry and corresponding Killing vector potential terms, which ensure the worldsheet supersymmetry [7]. The bosonic string action of a general class of pp-wave background supported by nonconstant R-R  $F_5$  flux, in light cone gauge, can be read off from the metric. The nonlinear sigma models have eight dimensional special holonomy manifold target space. The nonvanishing R-R fields gives, in particular, fermionic mass terms in the worldsheet action. Classical solutions of  $D$ -branes in pp-wave background with constant NS-NS and R-R flux are already discussed in the literature [10–17].  $Dp$ -branes from worldsheet point of view are constructed in [18]. Supersymmetric properties of  $D$ -branes and their bound states have also been analyzed both from supergravity and from worldsheet point of view.

$D$ -branes and their bound states play an important role in understanding various nonperturbative and duality aspects of string theory and gauge theories. The configurations of branes oriented by certain  $SU(N)$  angle are known to be supersymmetric objects [19–26]. They have also been useful in understanding the physics of black holes and gauge theories. So it is worth examining various classical solutions  $D$ -brane in plane wave spacetime as they also represent black holes in these backgrounds. The pp-wave spacetime with nonconstant five form flux has the interpretation of soliton solutions in two- dimensional sigma models as emphasized by Maldacena-Maoz [4]. (For a recent related work see [27]). So a natural extension would be to consider  $D$ -branes in these and in more general background to find out the interacting nonlinear sigma models on the worldsheet in the presence of  $D$ -branes. So it is desirable

to study various supergravity solutions of  $D$ -branes in order to have the spacetime realization of these objects and to study their supersymmetry properties as well.

In earlier work, we found some classical solutions of  $D$ -branes along with the supersymmetry in pp-wave spacetime with nonconstant NS-NS flux [28]. Intersecting  $D$ -branes in supergravities have also been discussed in [29, 30]. The possible black branes and the horizons have been discussed in the nonextremal deformations of  $D$ -branes in these backgrounds. So it is interesting to find out more  $D$ -brane solutions in plane wave spacetime with flux and to discuss the possibility of horizons in this framework. In this paper, we continue the search for supergravity brane solutions in plane wave spacetime with nonconstant R-R  $F_3$  flux. First we present the classical solutions of  $Dp$ -branes ( $p \geq 5$ ) in plane wave spacetime with nonconstant R-R  $F_3$  flux. Next, we find classical solution of a system of  $D3$ -branes oriented at an angle  $\alpha$ , ( $\alpha \in SU(2)$ ) with respect to each other in this background. In the  $D5$ -brane case all the worldvolume coordinates of the brane lie along the pp-wave directions and the transverse directions are flat. On the otherhand, for the  $D3$ -brane system only lightcone directions are along the brane, whereas the other pp-wave directions are along the transverse space. We would like to point out that the  $D$ -branes found in this paper are examples of localized  $D$ -branes in plane wave spacetime with flux. We would also like to point out that all the  $D$ -branes presented here are *longitudinal branes* as explained in [12]. The rest of the paper is organized as follows. In section-2, we present classical solutions of  $D$ -branes in pp-wave background with nonconstant R-R flux. Section-3 is devoted to the supersymmetry analysis of brane solutions presented in section-2. We conclude in section-4 with some discussions.

## 2 Supergravity Solutions

We start by writing down the supergravity solution of a system of  $D5$ -branes in the pp-wave background with non-constant R-R 3-form flux. The metric, dilaton and field strengths of such a configuration is given by:

$$\begin{aligned}
ds^2 &= f_5^{-\frac{1}{2}} \left( 2dx^+ dx^- + K(x_i)(dx^+)^2 + \sum_{i=1}^4 (dx_i)^2 \right) + f_5^{\frac{1}{2}} (dr^2 + r^2 d\Omega_3^2), \\
F &= \partial_1 b_2(x_i) dx^+ \wedge dx^1 \wedge dx^2 + \partial_3 b_4(x_i) dx^+ \wedge dx^3 \wedge dx^4, \\
e^{2\phi} &= f_5^{-1}, \quad F_{abc} = \epsilon_{abcd} \partial_d f_5, \\
f_5 &= 1 + \frac{N g_s l_s^2}{r^2},
\end{aligned} \tag{2.1}$$

with  $\square K(x_i) + (\partial_i b_j)^2 = 0$  and  $\square b(x_i) = 0$ .  $f_5$  denotes the harmonic function that satisfies Green function equation in the transverse 4-space. We have checked that the solution presented above satisfies all type IIB field equations. Other  $Dp$ -brane ( $p \geq 6$ ) solutions can be obtained by applying  $T$ -duality along  $x^5, \dots, x^8$  directions. For example: the  $D6$ -brane solutions, by applying  $T$ -duality along  $x^5$  (say), is given by:

$$\begin{aligned}
ds^2 &= f_6^{-\frac{1}{2}} \left( 2dx^+ dx^- + K(x_i)(dx^+)^2 + \sum_{i=1}^4 (dx^i)^2 + (dx^5)^2 \right) + f_6^{\frac{1}{2}} (dr^2 + r^2 d\Omega_2^2), \\
F &= \partial_1 b_2(x_i) dx^+ \wedge dx^1 \wedge dx^2 \wedge dx^5 + \partial_3 b_4(x_i) dx^+ \wedge dx^3 \wedge dx^4 \wedge dx^5, \\
e^{2\phi} &= f_6^{-\frac{3}{2}}, \quad F_{ab} = \epsilon_{abc} \partial_c f_6, \\
f_6 &= 1 + \frac{N g_s l_s}{r}.
\end{aligned} \tag{2.2}$$

Where  $f_6$  is the harmonic function that satisfies Green function equation in the transverse 3-space. Similarly, one can continue the above exercise for finding out supergravity solutions of the higher branes like  $D7$  as well. Bound states of  $D$ -branes can also be constructed by applying  $T$ -duality in the ‘delocalized’  $D$ -brane solutions as explained in [31, 32]. For example a  $D5 - D7$  bound state can be obtained from a  $D6$  solution and so on. We would like to point out that the solutions presented here are the generalization of the  $D$ -brane solutions found out in [10]. However, the crucial difference lies in the realization of supersymmetry, which will be discussed in the next section.

Now we present the classical solutions of a system of  $D3$ -branes oriented at an  $SU(2)$  angle with respect to each other in pp-wave background with nonconstant R-R 3-form flux. First, we present the supergravity solution of a single  $D3$ -brane oriented at an angle  $\alpha \in SU(2)$  with respect to the reference axis. To start with, the  $D3$ -brane is lying along  $x^+, x^-, x^6$  and  $x^8$  directions. By applying a rotation between  $(x^5 - x^6)$  and  $(x^7 - x^8)$ -planes following [22], with rotation angles  $(\alpha_1, \alpha_2) = (0, \alpha)$ , we get a configuration where the original  $D3$ -brane is tilted by an angle  $\alpha$ . In stead of going more into the constructional details, below we write down the classical solution of a single  $D3$ -brane rotated by an angle  $\alpha$ :

$$\begin{aligned}
ds^2 &= \sqrt{1 + X_1} \left\{ \frac{1}{1 + X_1} \left( 2dx^+ dx^- + K(x_i)(dx^+)^2 \right. \right. \\
&+ [1 + X_1 \cos^2 \alpha] [(dx^5)^2 + (dx^7)^2] + [1 + X_1 \sin^2 \alpha] [(dx^6)^2 + (dx^8)^2] \\
&+ \left. \left. 2X_1 \sin \alpha \cos \alpha (dx^7 dx^8 - dx^5 dx^6) \right) + \sum_{i=1}^4 (dx^i)^2 \right\}
\end{aligned}$$

$$\begin{aligned}
F &= \partial_1 b_2(x_i) dx^+ \wedge dx^1 \wedge dx^2 + \partial_3 b_4(x_i) dx^+ \wedge dx^3 \wedge dx^4, \\
F_{+-68i}^{(5)} &= -\frac{\partial_i X_1}{(1+X_1)^2} \cos^2 \alpha, \quad F_{+-67i}^{(5)} = \frac{\partial_i X_1}{(1+X_1)^2} \cos \alpha \sin \alpha, \\
F_{+-57i}^{(5)} &= \frac{\partial_i X_1}{(1+X_1)^2} \sin^2 \alpha, \quad F_{+-58i}^{(5)} = -\frac{\partial_i X_1}{(1+X_1)^2} \cos \alpha \sin \alpha, \\
e^{2\phi} &= 1.
\end{aligned} \tag{2.3}$$

and  $X_1$  is given by

$$X_1(\vec{r}) = \frac{1}{2} \left( \frac{\ell_1}{|\vec{r} - \vec{r}_1|} \right)^2. \tag{2.4}$$

Where  $r$  is the radius vector in the transverse space, defined by  $r^2 = \sum_{i=1}^4 (x^i)^2$ ,  $r_1$  is the location of  $D3$ -brane and  $X_1$  is the Harmonic function in the transverse space. One can easily check that the above ansatz solve type IIB field equations, with  $\square K(x_i) = -(\partial_i b_j)^2$  and  $\square b(x_i) = 0$ .

Next, we present the supergravity solution of a system of two  $D3$ -branes oriented at an angle  $\alpha$  with respect to each other. In this case, to start with two  $D3$ -branes are parallel to each other and are lying along  $x^+, x^-, x^6, x^8$  directions. By applying an  $SU(2)$  rotation as described earlier, the second brane rotated by an angle  $\alpha$ , now lies along  $x^+, x^-, x^5$  and  $x^7$  directions. The metric, dilaton and the field strengths of such a system is given by:

$$\begin{aligned}
ds^2 &= \sqrt{1+X} \left\{ \frac{1}{1+X} \left( 2dx^+ dx^- + K(x_i) (dx^+)^2 \right. \right. \\
&\quad + (1+X_2) [(dx^5)^2 + (dx^7)^2] + (dx^6)^2 + (dx^8)^2 \\
&\quad \left. \left. + X_1 [(\cos \alpha dx^5 - \sin \alpha dx^6)^2 + (\cos \alpha dx^7 + \sin \alpha dx^8)^2] \right) + \sum_{i=1}^4 (dx^i)^2 \right\}, \\
F &= \partial_1 b_2(x_i) dx^1 \wedge dx^2 + \partial_3 b_4(x_i) dx^3 \wedge dx^4, \\
F_{+-68i}^{(5)} &= \partial_i \left\{ \frac{X_2 + X_1 \cos^2 \alpha + X_1 X_2 \sin^2 \alpha}{(1+X)} \right\}, \\
F_{+-58i}^{(5)} &= -F_{+-67i}^{(5)} = \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{(1+X)} \right\}, \\
F_{+-57i}^{(5)} &= -\partial_i \left\{ \frac{(X_1 + X_1 X_2) \sin^2 \alpha}{(1+X)} \right\}, \quad e^{2\phi} = 1.
\end{aligned} \tag{2.5}$$

and  $X$  is the Harmonic function in the transverse space which is given by

$$X = X_1 + X_2 + X_1 X_2 \sin^2 \alpha, \quad (2.6)$$

where as defined earlier,  $X_{1,2} = \frac{1}{2} \left( \frac{\ell_{1,2}}{|\vec{r} - \vec{r}_{1,2}|} \right)^2$ . Once again we have checked that the above solution solve type IIB field equations, with  $\square K(x_i) = -(\partial_i b_j)^2$  and  $\square b(x_i) = 0$ . More  $D$ -brane bound states can be obtained by applying  $T$ -duality transformation along  $x^5, \dots, x^8$  directions. We would like to point out that the  $D$ -brane solutions presented here are the generalizations of the solutions presented in [16, 22].  $D$ -branes in plane wave background with nonconstant  $NS - NS$  flux can be obtained by applying  $S$ -duality on the above solutions. We, however, will skip those details. In the next section we will analyze the supersymmetry of these solutions by solving type IIB Killing spinor equations explicitly.

### 3 Supersymmetry Analysis

The supersymmetry variation of dilatino and gravitino fields of type IIB supergravity in ten dimension, in string frame, is given by [33, 34]:

$$\delta \lambda_{\pm} = \frac{1}{2} (\Gamma^{\mu} \partial_{\mu} \phi \mp \frac{1}{12} \Gamma^{\mu\nu\rho} H_{\mu\nu\rho}) \epsilon_{\pm} + \frac{1}{2} e^{\phi} (\pm \Gamma^M F_M^{(1)} + \frac{1}{12} \Gamma^{\mu\nu\rho} F_{\mu\nu\rho}^{(3)}) \epsilon_{\mp}, \quad (3.1)$$

$$\begin{aligned} \delta \Psi_{\mu}^{\pm} &= \left[ \partial_{\mu} + \frac{1}{4} (w_{\mu\hat{a}\hat{b}} \mp \frac{1}{2} H_{\mu\hat{a}\hat{b}}) \Gamma^{\hat{a}\hat{b}} \right] \epsilon_{\pm} \\ &+ \frac{1}{8} e^{\phi} \left[ \mp \Gamma^{\mu} F_{\mu}^{(1)} - \frac{1}{3!} \Gamma^{\mu\nu\rho} F_{\mu\nu\rho}^{(3)} \mp \frac{1}{2.5!} \Gamma^{\mu\nu\rho\alpha\beta} F_{\mu\nu\rho\alpha\beta}^{(5)} \right] \Gamma_{\mu} \epsilon_{\mp}, \end{aligned} \quad (3.2)$$

where we have used  $(\mu, \nu, \rho)$  to describe the ten dimensional space-time indices, and hat's represent the corresponding tangent space indices. Solving the above two equations for the  $D5$ -brane solution (2.1), we get several conditions on the spinors.

First the dilatino variation gives:

$$\Gamma^{\hat{a}} f_{5,\hat{a}} \epsilon_{\pm} + f_5^{-\frac{1}{4}} \Gamma^{\hat{i}\hat{j}} \partial_i b_j(x_i) \epsilon_{\mp} + \frac{1}{3!} \Gamma^{\hat{a}\hat{b}\hat{c}} \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}} f_{5,\hat{d}} \epsilon_{\mp} = 0. \quad (3.3)$$

On the other hand, the gravitino variation (3.2) gives the following conditions on the spinors:

$$\delta \psi_{+}^{\pm} \equiv \partial_{+} \epsilon_{\pm} + \frac{1}{4} f_5^{-\frac{1}{4}} \partial_i K(x_i) \Gamma^{\hat{i}} \epsilon_{\pm} - \frac{1}{8} f_5^{-\frac{1}{2}} \Gamma^{\hat{i}\hat{j}} \partial_i b_j(x_i) \Gamma^{\hat{c}} \epsilon_{\mp} = 0 \quad (3.4)$$

$$\delta \psi_{-}^{\pm} \equiv \partial_{-} \epsilon_{\pm} = 0 \quad (3.5)$$

$$\delta\psi_i^\pm \equiv \partial_i\epsilon_\pm - \frac{1}{8}f_5^{-\frac{1}{2}}\Gamma^{\hat{i}\hat{j}\hat{k}}\partial_{\hat{j}}b_{\hat{k}}(x_i)\delta_{i\hat{i}}\Gamma^{\hat{i}} = 0 \quad (3.6)$$

$$\delta\psi_a^\pm \equiv \partial_a\epsilon_\pm - \frac{1}{8}\frac{\partial_a f_5}{f_5}\epsilon_\pm - \frac{1}{8}\Gamma^{\hat{i}\hat{j}\hat{j}}\partial_{\hat{i}}b_{\hat{j}}(x_i)\delta_{a\hat{a}}\Gamma^{\hat{a}}\epsilon_\mp = 0 \quad (3.7)$$

In writing the above gravitino variation equations we have made use of the  $D5$ -brane supersymmetry condition:

$$\Gamma^{\hat{a}}\epsilon_\pm + \frac{1}{3!}\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}\Gamma^{\hat{b}\hat{c}\hat{d}}\epsilon_\mp = 0 \quad (3.8)$$

One notices that the supersymmetry condition (3.6), for nonconstant  $F_3$ :  $\partial_i\partial_{\hat{j}}b_{\hat{k}} \neq 0$ , can be satisfied only if  $\Gamma^{\hat{i}}\epsilon_\pm = 0$  [5].

Using  $\Gamma^{\hat{i}}\epsilon_\pm = 0$  and the brane supersymmetry condition (3.8), the dilatino variation (3.3) is satisfied. Now, the supersymmetry condition (3.7) is satisfied for the spinor  $\epsilon_\pm$ :  $\epsilon_\pm = \exp(-\frac{1}{8}\ln f_5)\epsilon_\pm^0$ , with  $\epsilon_\pm^0$  being a function of  $x^+$  only. Since  $\epsilon_\pm^0$  is independent of  $x^i$  and  $x^a$  whereas  $\partial_{\hat{i}}b_{\hat{j}}$  is a function of  $x^i$  only, from the gravitino variation (3.4), one gets the following conditions to have nontrivial solutions:

$$\partial_{\hat{i}}b_{\hat{j}}(x_i)\Gamma^{\hat{i}\hat{j}}\epsilon_\pm^0 = 0 \quad (3.9)$$

and

$$\partial_+\epsilon_\pm^0 = 0 \quad (3.10)$$

For the particular case when  $F_{+12} = F_{+34}$ , the equation (3.9) gives the following condition with constant spinor,  $\epsilon_\pm^0$ :

$$\Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}\epsilon_\pm^0 = \epsilon_\pm^0. \quad (3.11)$$

Therefore the  $D5$ -brane solution (2.1) preserves 1/8 supersymmetry.

Now we analyze the supersymmetry of the system of two  $D3$ -branes as presented in (2.5). The dilatino variation gives:

$$\Gamma^{\hat{i}\hat{j}\hat{j}}\partial_{\hat{i}}b_{\hat{j}}(x_i)\epsilon_\mp = 0. \quad (3.12)$$

The gravitino variation gives the following conditions on the spinors to be solved:

$$\delta\psi_+^\pm \equiv \partial_+\epsilon_\pm + \frac{1}{4}\partial_{\hat{i}}\left((1+X)^{-\frac{1}{4}}K(x_i)\right)\Gamma^{\hat{i}} - \frac{1}{8}\partial_{\hat{i}}b_{\hat{j}}(x_i)\Gamma^{\hat{i}\hat{j}}\Gamma^{\hat{a}}\epsilon_\mp$$

$$\begin{aligned}
& \mp \frac{1}{8} \Gamma^{\hat{+}\hat{-}\hat{6}\hat{8}\hat{i}} \left[ \frac{(1 + X_1 \sin^2 \alpha)^2 \partial_i X_2 + \cos^2 \alpha \partial_i X_1}{(1 + X)^{3/2} (1 + X_1 \sin^2 \alpha)} \right] \Gamma^{\hat{+}} \epsilon_{\mp} \\
& \mp \frac{1}{8} \Gamma^{\hat{+}\hat{-}\hat{5}\hat{7}\hat{i}} \left[ \frac{1}{(1 + X)^{5/2}} (1 + X_1 \sin^2 \alpha) \right. \\
& \times (X_1^2 \cos^2 \alpha \sin^2 \alpha \partial_i X_2 + (1 + X_2)^2 \sin^2 \alpha \partial_i X_1) \\
& - \frac{(X_1^2 \cos^2 \alpha \sin^2 \alpha) (1 + X_1 \sin^2 \alpha)^2 \partial_i X_2 + \cos^2 \alpha \partial_i X_1}{(1 + X)^{5/2} (1 + X_1 \sin^2 \alpha)} \\
& + \frac{1}{(1 + X)^{5/2}} \left( 2X_1^2 \cos^2 \alpha \sin^2 \alpha \partial_i X_2 + 2X_1^3 \cos^2 \alpha \sin^4 \alpha \partial_i X_1 \right. \\
& \left. - 2X_1 (1 + X_2) \cos^2 \alpha \sin^2 \alpha \partial_i X_1 \right) \left. \right] \Gamma^{\hat{+}} \epsilon_{\mp} \\
& \mp \frac{1}{8} \left\{ \frac{\Gamma^{\hat{+}\hat{-}\hat{5}\hat{8}\hat{i}} - \Gamma^{\hat{+}\hat{-}\hat{6}\hat{7}\hat{i}}}{(1 + X)^2 (1 + X_1 \sin^2 \alpha)} \right\} \left[ (-X_1 \cos \alpha \sin \alpha \partial_i X_2 \right. \\
& + (1 + X_2) \sin \alpha \cos \alpha \partial_i X_1 - X_1^2 \sin^3 \alpha \cos \alpha \partial_i X_1) (1 + X_1 \sin^2 \alpha) \\
& + X_1 \cos \alpha \sin \alpha (1 + X_1 \sin^2 \alpha)^2 \partial_i X_2 \\
& \left. + X_2 \cos^3 \alpha \sin \alpha \partial_i X_1 \right] \Gamma^{\hat{+}} \epsilon_{\mp} = 0 \tag{3.13}
\end{aligned}$$

$$\delta \psi_{\pm}^{\pm} \equiv \partial_{\pm} \epsilon_{\pm} = 0, \tag{3.14}$$

$$\delta \psi_a^{\pm} \equiv \partial_a \epsilon_{\pm} - \frac{1}{8} (1 + X)^{\frac{1}{4}} \partial_i b_j(x_i) \Gamma^{\hat{+}\hat{i}\hat{j}} \Gamma_a \epsilon_{\mp} = 0, \quad (a = 5, \dots, 8), \tag{3.15}$$

$$\delta \psi_i^{\pm} \equiv \partial_i \epsilon_{\pm} - \frac{1}{8} \frac{\partial_i X}{(1 + X)} \epsilon_{\pm} - \frac{1}{8} (1 + X)^{\frac{1}{2}} \partial_j b_k(x_i) \Gamma^{\hat{+}\hat{j}\hat{k}} \delta_{i\hat{i}} \Gamma^{\hat{i}} \epsilon_{\mp} = 0. \tag{3.16}$$

In writing down the above supersymmetry variations, we have made use of the following conditions [16]:

$$(\Gamma^{\hat{5}\hat{8}} - \Gamma^{\hat{6}\hat{7}}) \epsilon_{\mp} = 0, \quad (\Gamma^{\hat{5}\hat{7}} + \Gamma^{\hat{6}\hat{8}}) \epsilon_{\mp} = 0, \tag{3.17}$$

$$\Gamma^{\hat{+}\hat{-}\hat{6}\hat{8}} \epsilon_{\mp} = \epsilon_{\pm}, \quad \Gamma^{\hat{+}\hat{-}\hat{5}\hat{7}} \epsilon_{\mp} = \epsilon_{\pm}. \tag{3.18}$$

To explain further, the conditions written in (3.17) comes from the rotation between the two  $D3$ -branes and those in (3.18) are the  $D3$ -brane supersymmetry conditions.



It is rather straightforward to conclude the conditions written in eqns. (3.17) and (3.18) are infact two independent conditions, thereby breaking 1/4 supersymmetry. As explained earlier, the equation (3.16), for nonconstant  $\partial_{\hat{j}} b_{\hat{k}}$ , can be solved by the spinor  $\epsilon_{\pm}$ :  $\epsilon_{\pm} = \exp(-\frac{1}{8} \ln(1 + X)) \epsilon_{\pm}^0$ , with  $\epsilon_{\pm}^0$  being a function of  $x^+$ , only if:

$$\Gamma^{\hat{+}} \epsilon_{\pm} = 0. \quad (3.19)$$

Now putting the condition (3.19), the dilatino variation is satisfied. All the gravitino variations are also satisfied leaving the following two equations to have nontrivial solutions.

$$\partial_{\hat{i}} b_{\hat{j}}(x_i) \Gamma^{\hat{i}\hat{j}} \epsilon_{\mp}^0 = 0. \quad (3.20)$$

and

$$\partial_+ \epsilon_{\pm}^0 = 0, \quad (3.21)$$

Once again for the particular case  $F_{+12} = F_{+34}$ , eqn.(3.20) gives:  $(1 - \Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}) \epsilon_{\mp}^0 = 0$  for constant spinor,  $\epsilon_{\pm}^0$ . Therefore the system of  $D3$ -branes (2.5) preserves 1/16 supersymmetry [16].

## 4 Summary and Discussion

In this paper we have constructed various localized  $D$ -brane configurations in plane wave spacetime with nonconstant R-R 3-form flux. The supersymmetry of these branes have been analyzed by solving type IIB Killing spinor equations explicitly. The existence of other  $Dp$ -brane ( $p < 5$ ) solutions in this plane wave spacetime puts restriction on the localization of the branes and also on the behaviour of function  $K(x_i)$  parameterizing the plane wave spacetime [13, 35]. The  $\mathcal{H}$ -deformed  $D$ -branes can also be constructed following [29, 30, 36]. Though the nonextremal  $D$ -branes admit horizons and known as black branes, this is not in general true in plane wave spacetime [29, 30, 37]. One could possibly look at the black brane solutions in this background and discuss properties of their horizon.

The worldsheet construction of  $D5$ -brane and the corresponding nonlinear sigma model of the background considered in this paper can be found out by refering to the following Green-Schwarz action [5] written in lightcone gauge and the  $D5$ -brane boundary condition:

$$L_B = \partial_+ x_i \partial_- x_i - \frac{1}{2} m^2 b_i^2 + \partial_+ y_a \partial_- y_a, \quad (4.1)$$

$$L_f = i\theta_R\gamma^v\partial_+\theta_R + i\theta_L\gamma^v\partial_-\theta_L - \frac{1}{4}im\partial_ib_j(x_i)\theta_L\gamma^v\gamma^{ij}\theta_R, \quad (4.2)$$

$$m \equiv \alpha' p^u = \partial_\pm u, \quad (4.3)$$

where  $\theta_L$  and  $\theta_R$  are the majorana-Weyl spinors in the left and right moving sectors and  $x^i$ , ( $i = 1, \dots, 4$ ) and  $y_a$ , ( $a = 5, \dots, 8$ ) denote the worldvolume and transverse directions of the  $D5$ -brane respectively. The plane wave background with nonconstant R-R flux can also be parametrized by holomorphic function on the worldsheet [5]. So it is useful to analyze the interacting Lagrangian in the presence of these nonperturbative objects. The conditions of consistent  $D$ -brane which were obtained in [6] are expected to be different in the present case because of the flat transverse space. So an interesting exercise will be to obtain all the consistent  $D$ -branes of [6]. That would probably tell us about the integrability structure of the worldsheet theory in the presence of branes, if it works out nicely, in a more general background. A systematic classification of all supersymmetric  $D$ -branes from worldvolume point of view is also needed. Finally, it would really be nice to find out the holographic dual of these plane wave backgrounds in the presence of branes. We hope to come back to these issues in future.

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